ABSTRACT

of the dissertation work of Kassenova Tolkyn Kalmakhanovna on the topic of «Investigation of the connection of knot theory with models of statistical mechanics», submitted for the degree of Doctor of Philosophy (PhD) in the specialty: "6D060400 - Physics"

The relevance of the research topic

The connection between knot theory and statistical mechanics became popular after scientists put forward the possibility of tying water into knots of vortex flows, which takes place in real physical objects. This finding allows us to study the modular system, which allows us to study in detail the independent resolution of knots in real physical processes. Later, it was proved that tying water into knots makes it possible to understand the theories of superconductivity, fluid superfluidity and the action of liquid crystals.

The relevance of the thesis lies in the fact that the main problems of knot theory remain open: attempts to strictly classify knots have not yet been solved, and it is not yet known whether they have a computably complete system of invariants; knots are used both in pure mathematics and in real physical, chemical and biological processes and objects.

The need to study the connection between the theory of knots and models of statistical mechanics are:

- firstly, the statistical sum depends only on the sum over all possible states of the energy function of the spin model. This means that each state determines the spins of all vertices of the graph and thus the energy of interactions between them. To calculate the statistical sum, you need to assign to each edge of the graph an exponent of the interaction energy of the vertices connected by this edge. This process is then repeated for all possible states;

- secondly, the energy functions of models satisfying the star-triangle relations (and other relations corresponding to other Reidemeister moves) will give statistical sums that are also engagement invariants.

The existing algebraic structures of solvable lattice models, reformulated as an Ising-type model with only two-spin interaction, do not sufficiently allow us to determine a complete list of knot invariants.

The beginning of the development of the theory of knots was laid by the great English physicist J. Maxwell believed that waves interact in an electromagnetic way, and then put forward an even bolder idea: interacting particles are waves in themselves; but since the particles (atoms) are so small and the waves are so long, wave atoms should be limited to a small area of space: they are encoded knots that store all the physical and chemical information about the nature of the atomic lattice.

This theory requires solving a number of problems relating spin models of the ice type of statistical mechanics with knot invariants through braid theory and the Wess-Zumino conformal field theory.

Thus, the study of the above scientists allows us to consider the polynomial invariants of the theory of knots, their structures and their special significance for topology and their relationship with mathematical physics. The relationship with physics begins with the connection between the main models for knot polynomials and statistical functions. This relationship leads to the construction of a number of invariants that go beyond the original knot polynomials. Also a key problem is the question of the differentiation of new invariants.

In addition to these and many other applications of knot theory in topology, its applications also include the study of the features of plane algebraic curves, and in a multidimensional situation - isolated features of complex hyper surfaces, smooth structures on spheres, the construction of dynamical systems and foliations.

The connection between the knot theory and the vertex models of statistical mechanics remains alive and mysterious. The main questions are still open today, and finally, it is believed that the leading role of knot theory in statistical physics has not yet been definitively determined.

The goal of the research is to study the connection between knot theory invariants and partition functions in problems of statistical physics.

To achieve this goal, it is necessary to solve the following tasks:

1 For a ten-vertex model of statistical mechanics, construct representations of generators of the braid group leading to the calculation of the knot invariant;

2 Generalize the procedure for determining representations of the braid group generators from the R-matrix associated with the three-particle vertex model and derive the algebraic formula for the knot invariant.

3 Obtain new solutions of eight-vertex and six-vertex models of statistical mechanics on a square lattice over Grassmann algebra using the tetrahedron equation, which is a three-dimensional generalization of the well-known Yang-Baxter equation;

4 Classify the solutions of the Yang-Baxter equation for vertex models and study them.

5 Develop a method that provides a general way to construct new integrable nearest neighbor models. On the basis of integrability, determine the exact value of the eigenstate of the matrix in terms of the algebraic Bethe ansatz;

6 Calculate the values of the elements of the ten-vertex model of statistical mechanics with half-integer spin. Find the eigenstate and eigenvalue of the transfer matrix using the Yang–Baxter equation.

The objects of the research are topological model of knot theory and vertex models of statistical mechanics.

The subject of the research is connection of the three-dimensional representation of knots in the topological model of the theory of knots with the elements of the quantum monodromy matrix (Yang-Baxter equations).

Research methods: methods of statistical physics, knot theory, braid theory and group theory.

The main statements for defense:

1. Under the condition of fermion-boson interaction using the SU (2) Chern-Simons theory, the Hamiltonian of the three-particle vertex model of statistical mechanics is expressed in terms of the Jones polynomial of knot theory.

2. Under periodic boundary conditions with domain walls, generators obeying the Pauli exclusion principle generate new solutions of the Zamolodchikov equation in eight-vertex and six-vertex integrable models of statistical mechanics in three-dimensional space.

3. Under conditions of asymmetrization using the limit of spectral parameters - matrix elements, the partition function of the ten-vertex model of statistical mechanics is described by the Jones polynomial.

Scientific novelty of the work. lies in the fact that for the first time a three-particle vertex model of statistical mechanics has been developed and the knot invariant has been calculated for the first time. For the six-vertex and eight-vertex models of statistical physics, new solutions of the three-dimensional generalization of the Yang-Baxter equation are obtained. For the first time, an exactly solvable ten-vertex model of statistical mechanics is considered and the corresponding knot polynomial is found.

The theoretical and practical significance of the research outcomes.

The dissertation work is theoretical in nature. Its results can be used to study meshing, graph theory and braids, and to explain a family of vertex models for crystal lattices with hydrogen bonds, as well as models of some ferroelectric and antiferroelectric crystals. The final result of this work can later be used to study invariants of knots used in biology, statistical mechanics and chemistry.

Also, the results of the dissertation work can be applied in the educational process for reading elective courses to bachelors, undergraduates and doctoral students of the specialty "Physics".

The validity and reliability of the research results obtained is confirmed by publications in peer-reviewed scientific journals and participation in international conferences. The achieved scientific results are in good agreement with the existing works of other foreign authors in this field.

Personal contribution of the author.

The author participated in determining the purpose of the work and setting research objectives, actively participated in discussing the results of the dissertation and writing articles. The author took part in the discussion of the results of dissertation research and the formulation of conclusions. The scientific articles published based on the results of the dissertation were written in collaboration with the supervisor.

Approbation of the dissertation. The materials of the dissertation work were reported at the following republican and international conferences:

- 8th International Conference on Mathematical Modeling in Physical Science Journal of Physics: Conference Series. Eight-vertex model over Grassmann algebra.- 2019. -№1391, 012035. IOP Publishing. doi 10.1088/1742-6596/1391/1/0120351.

- XIV International Scientific Conference of Students and Young scientists "Science and Education – 2019". An exact solution for an eight-vertex spin model. -2019. - Section 1. - P. 272-276.

- The XVI International Scientific Conference of students, undergraduates and Young scientists "Lomonosov - 2020". Abstracts of reports 1 part. - 2020. - P. 29-30.

- Proceedings of the IV International Conference "Astrophysics, Gravity and Cosmology". Application Grassmann algebra for a vertex model. - 2019. - p.47.

In addition, the results were reported and discussed at scientific seminars of the Department of General and Theoretical Physics of L.N. Gumilyov ENU, at seminars of the Eurasian International Center for Theoretical Physics and seminars of the Department of Physics of California State University, Fresno.

Publications.

Based on the materials of the dissertation, 8 scientific papers have been published in Kazakhstani and foreign journals, including: 1 article in a foreign journal with a high impact factor, included in the Web of Science database and Scopus; 3 articles in periodicals recommended by the Committee for Control in the Field of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan, including 1 article in the cited journal in the database Web of Science and Scopus (Corresponding author); 1 article in a foreign international conference, included in the Web of Science and Scopus database (Corresponding author); 2 articles and 1 thesis in the materials of international conferences (Corresponding author).

The connection of the topic of the dissertation with the plans of scientific works. The work was carried out in accordance with the plans of research work on the project - «The study of some problems of astrophysics and cosmology in the framework of Einstein and non–Einstein theories of gravity». The project implementation years are 01.01.2018 - 01.01.2020. IRN of the project BR05236277.

Head of the IRN project No. AP14972644 «Investigation of the connection between exactly solvable vertex models of statistical mechanics and knot invariants» for grant funding of fundamental and applied scientific research of young postdoctoral scientists under the Zhas Galym project for 2022-2024 in priority areas of science development.

The structure and volume of the thesis. The dissertation consists of an introduction, three sections, a conclusion and a list of references. The volume of the dissertation is 94 pages, containing 1 figures and 2 tables, the number of used literature sources is 161.